Benchmark problem: Direct Simulation Monte Carlo of gas flow through an orifice and short tube

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Outline

1. Statement of the problem
2. Previous theoretical & experimental work
3. Method of solution - Computational grid - Numerical parameters
4. Numerical results - Comparisons with experimental results
5. Concluding remarks
Statement of the problem

• The reduced flow rate:

\[ W = \frac{\text{mass flow rate}}{\text{mass flow rate for } \delta=0 \text{ and } P_1 = 0} = \frac{\dot{M}}{\dot{M}_0} = \frac{\dot{M}}{\sqrt{\frac{\pi}{3}} R^2 P_0} \]

• Flow field

\[ \delta = \frac{P_0 R}{\mu v_0} \]

• Rarefaction parameter

\[ v_0 = \sqrt{\frac{2 k_B T_0}{m}} \]

• Most probable velocity
A 3-level computational grid is used in order to increase the resolution near the walls.

Several weighting zones are used in the axial and radial direction to avoid non-uniform distribution of the model particles.
Previous theoretical & experimental work

Orifice flow \((L/R=0)\)


Short tubes \((L/R\neq0)\)

Method of solution

- The DSMC method was applied, based on:
  - Hard sphere model (HS, VHS)
  - Purely diffuse gas - surface interaction or C-L
  - No Time Counter (NTC) scheme for the simulation of the intermolecular collisions

- Range of flow and geometrical parameters:
  - Pressure ratio: \( \frac{P_0}{P_1} = 0 \sim 0.9 \)
  - Dimensionless length: \( 0 \leq L/R \leq 10 \)
  - Rarefaction parameter: \( 0 \leq \delta \leq 1000 \)
Numerical parameters

- Number of model particles: $30-40 \times 10^6$.
- Number of samples: $10^4-10^5$.
- Size of largest cells: $\Delta x = \Delta r = R/20$.
- Time increment: $\Delta t = 0.01(R/\nu_0)$.
- Size of computational region: $L_1 = R_1 = 8R$ and $L_2 = R_2 = 8R$.
- The variance of $W$ is less than 1%.

Computational effort

- Depending on flow and geometrical parameters, the CPU time ranges from one day to several weeks.
- Memory: 1-1.5 Gb
# Numerical results for the reduced flow rate $W$

<table>
<thead>
<tr>
<th></th>
<th>$L/R=0^a$</th>
<th>$L/R=1$</th>
<th>$L/R=5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>$p_1/p_0=0$</td>
<td>$p_1/p_0=0.5$</td>
<td>$p_1/p_0=0^b$</td>
</tr>
<tr>
<td>0.01</td>
<td>1.001</td>
<td>0.673</td>
<td>0.673</td>
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<tr>
<td>0.1</td>
<td>1.014</td>
<td>0.680</td>
<td>0.680</td>
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<tr>
<td>1</td>
<td>1.129</td>
<td>0.754</td>
<td>0.754</td>
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<tr>
<td>10</td>
<td>1.462</td>
<td>1.062</td>
<td>1.062</td>
</tr>
<tr>
<td>100</td>
<td>1.534</td>
<td>1.358</td>
<td>1.358</td>
</tr>
</tbody>
</table>


Numerical results for the reduced flow rate $W$

Comparison with experimental results (1)

Comparison with experimental results (2)

Short tube with $L/R=2$, Nitrogen, ambient $T$. 

Short tube with $L/R=8.56$, Nitrogen, ambient $T$. 

Varoutis et. al., *FED*, 85, 1798–1802 (2010)
Flow field: $L/R=1$, $P_1/P_0=0.1$, $\delta=1$

Local Mach number

\[ \frac{n}{n_0} \]

\[ \frac{T}{T_0} \]
Flow field: \( L/R=1, \ P_1/P_0=0.1, \ \delta=1000 \)

Local Mach number

\[ \frac{n}{n_0} \]

\[ \frac{T}{T_0} \]
Flow field: $L/R=1$, $P_1/P_0=0.5$, $\delta=1$

Local Mach number

$n/n_0$

$T/T_0$
Flow field: $L/R=1$, $P_1/P_0=0.5$, $\delta=1000$

Local Mach number

$n/n_0$

$T/T_0$
Velocity profile: $L/R=0.5$, $P_1/P_0=0.1$

Velocity profiles at the inlet ($x=0$: _), middle ($x=L/(2R)$: — — —) and outlet ($x=L/R$: ———) cross sections of a tube with $L/R=0.5$. 
Velocity profile: $L/R=0.5, P_1/P_0=0.7$

Velocity profiles at the inlet ($x=0$: _), middle ($x=L/(2R)$: -- ) and outlet ($x=L/R$: ----) cross sections of a tube with $L/R=0.5$. 
Concluding remarks

- The flow of rarefied gas through an orifice and short tubes into a background gas has been investigated in the whole range of the rarefaction by the DSMC method.

- Implementation of a 3 level grid for improving the resolution near the stationary boundaries. The concept of weighting zones for achieving uniform distribution of particles, is introduced.

- Results including the reduced flow rate and distributions of macroscopic quantities of practical interest as for instance the local Mach number, pressure, temperature etc, have been reported.

- The dependency of these quantities on the length to radius ratio (L/R), the rarefaction parameter (δ) and the pressure ratio (P_1/P_0) has been examined.
Questions?