Rarefied gas flow through tubes of finite length

1 Statement of the problem

Consider an axisymmetrical flow of monoatomic gas through a tube of radius R and length L, connecting two semi-infinite reservoirs as is shown in Figure 1. The pressures in the left and right reservoirs far from the channel entrances are maintained equal to p_1 and p_2, respectively. The temperature of all walls and of the gas far from the channel entrances is the same and equal to T_1. The gas-surface interaction is assumed to be diffuse.

The aim is to calculate the mass flow rate $\dot{M}$ through the tube.

2 Input data

The problem is determined by three parameters:

(i) Gas rarefaction δ defined as

$$\delta = \frac{p_1 R}{\mu_1 v_m}, \qquad v_m = \sqrt{\frac{2kT_1}{m}}, \tag{1}$$

where μ_1 is the gas viscosity at the temperature T_1, v_m is the most probable speed at the same temperature, $k = 1.3806503 \times 10^{-23}$ J/K is the Boltzmann constant, and m is the mass of one molecule in kg.

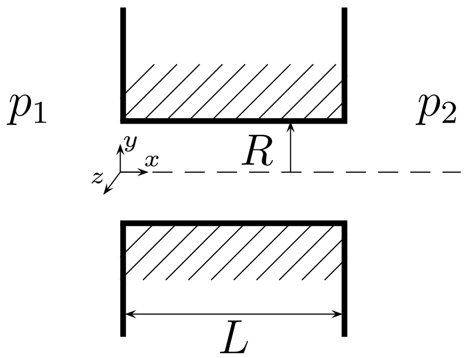

Figure 1: Scheme of the flow

Table 1: List of cases to be calculated

	δ	p_2/p_1	L/R		δ	p_2/p_1	L/R
1	0.01	0	0	11	0.01	0	1
2	0.1	0	0	12	0.1	0	1
3	1	0	0	13	1	0	1
4	10	0	0	14	10	0	1
5	100	0	0	15	100	0	1
6	0.01	0.5	0	16	0.01	0.5	1
7	0.1	0.5	0	17	0.1	0.5	1
8	1	0.5	0	18	1	0.5	1
9	10	0.5	0	19	10	0.5	1
10	100	0.5	0	20	100	0.5	1

(ii) Pressure ratio p_1/p_2.

(iii) Aspect ratio L/R.

It is suggested to solve the problem for 20 cases of the input parameters given in Table 1.

If the temperature T_1 must be specified, it is assumed to be 300 K.

If a species of the gas must be specified, it is assumed to be helium.

3 Output data

The output data should be provided in terms of the reduced flow rate defined as

$$W = \frac{\dot{M}}{\dot{M}_0}. \tag{2}$$

Here, $\dot{M}$ is the mass flow rate through at any δ, p_2/p_1, and L/R, while $\dot{M}_0$ is the mass flow rate at $\delta = 0$, $p_2/p_1 = 0$, and $L/R = 0$, i.e.

$$\dot{M}_0 = \frac{\sqrt{\pi}R^2 p_1}{v_m}. \tag{3}$$

4 Report

The report should contain the flow rate W for all or for some cases given in Table 1, method of solution, numerical uncertainty, CPU time.