UNIVERSITY OF THESSALY
SCHOOL OF ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

RAREFIED GAS FLOWS THROUGH SLITS AND ORIFICES

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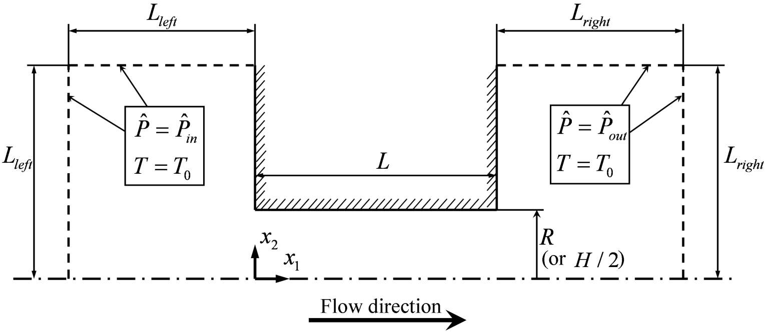
Introduction

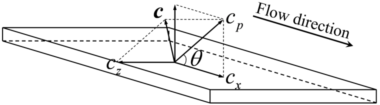


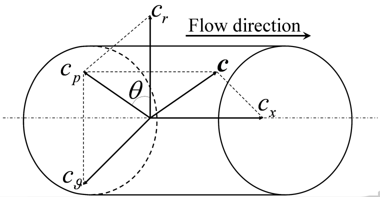
- Flow through slits ($L/H = 0$), orifices ($L/R = 0$) and short channels/tubes ($L/H = 1$, $L/R = 1$) has been studied
- The pressure ratio $P_{out} = \hat{P}_{out}/\hat{P}_{in}$ and the Knudsen number (or the rarefaction parameter $\delta \approx 1/Kn$) may take any value

Method of computation

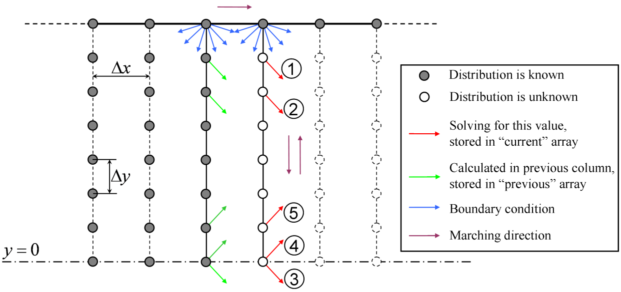
- The current problems have been solved discretizing the velocity space by the DVM and the physical space by a second order finite difference scheme.
- With this set of parameters our results have converged to at least 2-3 significant figures

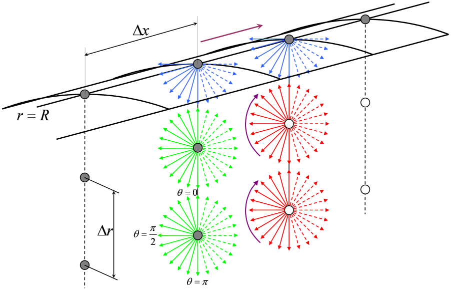
Parameters	Channel	Tube
Nodes / unit length ($N_x = N_y = N_r$)	100	100
Discrete angles (N_θ) in $(0, 2\pi)$	200	≥ 200
Discrete magnitudes M	20	16×16
Max. value of velocity magnitude (c_{max})	5	5
Convergence criterion	10^{-8}	10^{-8}
Container regions	$\geq 15 \times 15$	15×15



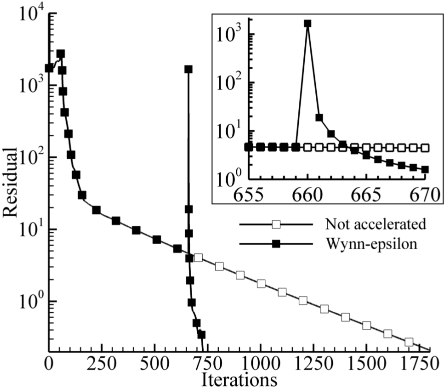


Marching scheme



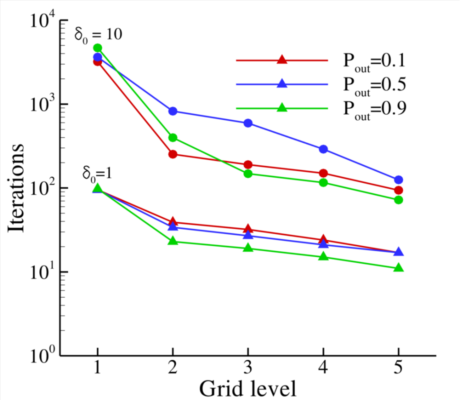


Wynn-ε algorithm



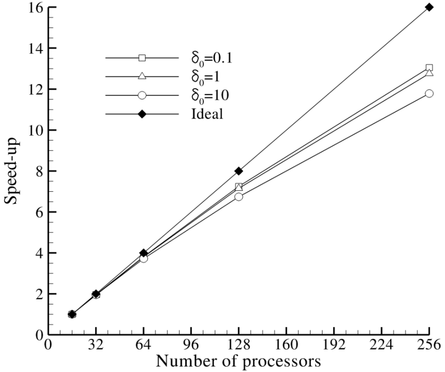
- All macroscopic quantities and impermeability constants are treated as sequences and the W-ε is used to accelerate their convergence in regular intervals

Grid refinement



- The simulations are initially performed in sparse grids and these solutions are interpolated and used as initial conditions in the next grid level

Parallelization



- Each molecular velocity magnitude (and angle in the 2D case) is independent from the rest, leading to a straightforward parallelization of the algorithm

Balance equations and analytical solutions

- Mass balance is fulfilled with very good accuracy for both geometries

$$\frac{\partial(\rho u_x)}{\partial x} + \frac{\partial(\rho u_y)}{\partial y} = 0$$

$$\frac{\partial(\rho u_x)}{\partial x} + \frac{\partial(\rho u_r)}{\partial r} = 0$$

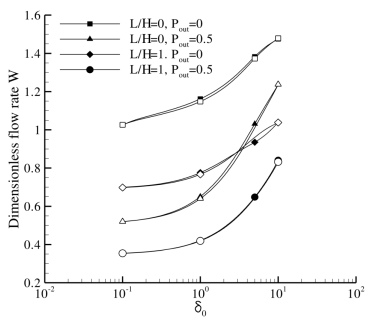
- The non-conservative form of equations have been used but the angular grid is dense enough to achieve the conservation principles
- Analytical solutions are recovered in the free molecular regime

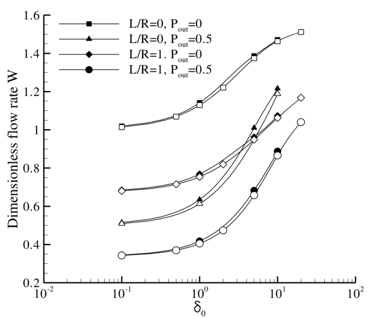
Kinetic models

δ	P_{out}	$L/R=0$			$L/R=1$		
		BGK	S	ES	BGK	S	ES
0.1	0.1	0.920	0.921	0.923	0.617	0.617	0.619
	0.5	0.515	0.516	0.518	0.345	0.346	0.347
	0.9	0.104	0.104	0.105	0.0695	0.0697	0.0700
1	0.1	1.06	1.06	1.07	0.703	0.706	0.713
	0.5	0.635	0.641	0.653	0.417	0.422	0.428
	0.9	0.135	0.137	0.140	0.0881	0.0894	0.0908
10	0.1	1.44	1.44	1.45	1.06	1.06	1.06
	0.5	1.21	1.21	1.22	0.885	0.886	0.888
	0.9	0.424	0.427	0.432	0.260	0.262	0.264

- Three kinetic models, namely the BGK, Shakhov and Ellipsoidal model have been applied
- Results are generally close to each other

Comparison with DSMC





Very good agreement has been found for both channel and tube flows

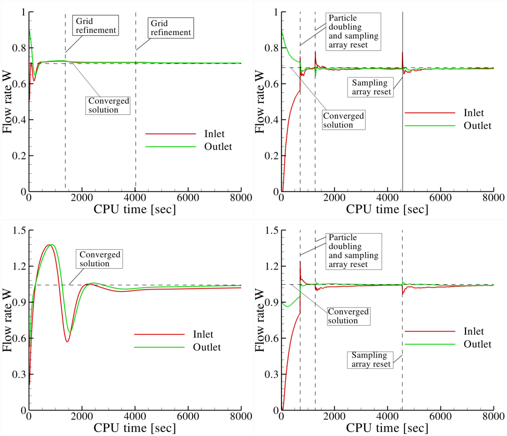
Comparison with linearized results

- Flow rates have been compared with corresponding results obtained by the linearized formulation for a pressure ratio $P_{out} = 0.9$
- The agreement is excellent

Slit		
δ	NL BGK	Lin. BGK [1]
0	0.100	0.0999
0.01	0.101	0.100
0.1	0.105	0.105
0.5	-	0.121
1	0.138	0.140
2	-	0.173
5	0.260	0.265
10	0.391	0.397

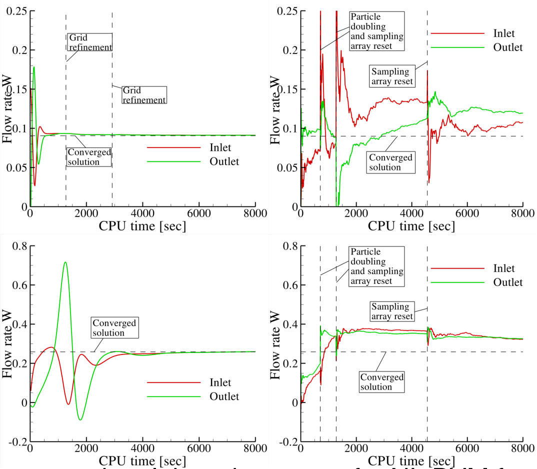
Orifice		
δ	NL BGK	Lin. BGK [1]
0	0.100	0.0999
0.1	0.104	0.104
1	0.135	0.137
10	0.424	0.435

Computational time - $P_{out} = 0.1$



- It is hard to provide a definite conclusion if the DSMC or the DVM perform better.
- In a qualitative comparison, performance is comparable to DSMC for large pressure differences

Computational time - $P_{out} = 0.9$



- The computational time decreases for NL DVM for small pressure differences
- The computational time increases for DSMC due to the high statistical noise

Memory consumption

Parameter values	Test case	Parameter dependence				
		L/R	N_θ	M	L_{left}	N_x
Length ratio L/R	10	200	10	10	10	10
Nodes / unit length $N_x = N_r$	100	100	100	100	100	120
Disc. angles N_θ in $[0, \pi]$	160	160	400	160	160	160
Disc. magnitudes $M_x = M_p$	16	16	16	32	16	16
Container size $L_{left} = L_{right}$	15	15	15	15	10	15
Memory consumption [MB]	922	1300	933	922	423	1300

- The proposed storage scheme is very beneficial for the memory requirements of the code
- Channels of any length can be simulated with an arbitrary discretization in velocity space
- Although it seems that there are some limitations in the number of nodes and the container size, it turns out that this is a minor restriction considering the availability of memory today

Concluding Remarks

- The problems of flow through channels and tubes, including slits and orifices, have been simulated by various kinetic models using the DVM
- Results have been validated in several manners, including balance equations, analytical solutions and linearized solutions
- Computational performance has been improved by Wynn-ε acceleration, parallelization and grid refinement techniques
- Good agreement is found between the BGK, S and ES models, as well as with the DSMC results.
- The algorithm is computationally efficient and can be used for more demanding geometries in the whole range of Knudsen
- The range of applicability of the linearized solutions is beyond its theoretical restrictions